

The chirality theorem

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Abstract

We show how chirality of the weak interactions stems from string independence in the string-local formalism of quantum field theory.

Σωκράτης – ὁ νοῦν ἔχων γεωργός, ὧν σπερμάτων κήδοιτο καὶ ἔγκαρπα
βούλοιο γενέσθαι, πόττερα σπουδῇ ἂν θέρους εἰς Ἀδώνιδος κήπους ἀρῶν...;
– Plato (*Phaidros*, 276b) [1]

1 Introduction

Unanswered questions abound in electroweak theory [2]. Only time will tell which ones were prescient, and which born only from theoretical prejudice [3]. An important trait of flavourdynamics is the chiral character of the interactions in which fermions and the massive vector bosons participate. A literature search shows that most textbooks dispatch this trait in one word: it is a *fact*. There are a few exceptions. The book by Peskin and Schroeder discusses at some length how left-handed and right-handed components of fermions can come to see (representations of, if you wish) different gauge groups [4, Chap. 19]. The posthumous, reflective book by Bob Marshak [5, Chaps. 1 and 6], discoverer (together with E. C. G. Sudarshan) of the Vector-Axial theory, interestingly elevates the “fact” to a principle, that of chirality invariance, or “neutrino paradigm”.

Nevertheless, on the face of it, there is a mystery here, setting flavourdynamics apart from chromodynamics. That cannot be solved by invoking the Glashow–Weinberg–Salam (GWS) model, which introduces chirality by hand from the outset.

The aim of this paper is to show that the physical particle spectrum of the interaction-carriers in the electroweak sector, including the scalar particle, *forces* the couplings of the

massive bosons to fermions to be parity-violating – and precisely how that is achieved. The proof, rigorous within perturbation theory, is given within the conceptual framework of string-local fields (SLF), without any appeal to gauge theory whatsoever.

To repeat: the charge and mass structure of the electroweak bosons contain all the information required to determine their relative coupling strengths with the fermion sector entirely. In particular, the weak interactions must be chiral. The argument is startlingly simple, in that it needs only consideration of tree graphs up to second order. Going *a posteriori* from our stuff to the GWS model for fermions is both trivial and almost inconsequential; nevertheless, we indicate how to do it in an appendix.

The plan of the article is as follows. Section 2 is a précis on free string-local fields. Section 3 introduces the simple principle of physical string-independence governing SLF couplings. Section 4 reviews the basics of perturbation theory and Epstein–Glaser renormalization, as adapted to SLF. The next section examines constraints imposed on couplings with fermions by string independence already at the first-order level.

Section 6 displays a method, due to one of us, to construct time-ordered products involving SLF for tree diagrams at second order. Once the above has been digested, the rest of the proof reduces to a series of straightforward calculations, performed in Section 7. Section 8 is the conclusion.

The appendices deal with a few relevant side questions. Appendix A comments on integration of quantum fields over lines. Appendices B and C provide some computational details. Appendix D provides proof of locality for our “stringy” fields. Appendix E manufactures the GWS model from the uncovered chiral coupling constants.

2 String-local fields

To define our SLF, we start from free Faraday tensor fields on Minkowski space \mathbb{M}_4 . These can be built from Wigner’s spin 1 or helicity ± 1 unitary, irreducible representations of the restricted Poincaré group [6], by use of appropriate creation operators $\alpha_r^\dagger(p)$ and polarization dreibein or zweibein $e_r^\mu(p)$, under the form:

$$F_a^{\mu\nu}(x) := \sum_r \int d\mu(p) \left[e^{i(p x)} (ip^\mu e_r^\nu(p) - ip^\nu e_r^\mu(p)) \alpha_{r,a}^\dagger(p) + e^{-i(p x)} (-ip^\mu e_r^\nu(p)^* + ip^\nu e_r^\mu(p)^*) \alpha_{r,a}(p) \right], \quad (2.1)$$

where $d\mu(p) := (2\pi)^{-3/2} d^3\mathbf{p}/2E(\mathbf{p})$. Such fields are of the Lorentz transformation type $(1,0) \oplus (0,1)$ – see [7, Sect. 5.6]. Consult also [8] in this respect. We use the notation $(ab) := g_{\lambda\kappa} a^\lambda b^\kappa = a^0 b^0 - \mathbf{a} \cdot \mathbf{b}$ for Minkowski inner products.

The set $\{F_a\}$ above includes one such field for each of the physical particles, universally denoted W^\pm, Z, γ . String-local “potential” fields for them are determined from the F_a :

$$A_a^\mu(x, l) := \int_0^\infty dt F_a^{\mu\lambda}(x + tl) l_\lambda, \quad (2.2)$$

with $l = (l^0, \mathbf{l})$ a null vector. By [half-]string we understand the set of points $\{x + tl\}$, with $t \geq 0$. Each of the A_a lives on the same Fock space as F_a .

Improving on old proposals by Mandelstam [9] and Steinmann [10], SLF theory was introduced in [11, 12]. It has found in Schroer an indefatigable apostle [13, 14].¹

The main properties of these SL fields are as follows:

- ★ Transversality: $(lA_a(x, l)) = 0$; and $(\partial A_a(x, l)) = 0$ in the massless boson case.²
- ★ Pointlike differential: $\partial^\mu A_a^\lambda(x, l) - \partial^\lambda A_a^\mu(x, l) = F_a^{\mu\lambda}(x)$, or $dA_a = F_a$ for short.
- ★ Covariance: let U denote the second quantization of the mentioned unitary representations of the restricted Poincaré group on the one-particle states. Then

$$U(a, \Lambda)A_a^\mu(x, l)U^\dagger(a, \Lambda) = A^\lambda(\Lambda x + a, \Lambda l)\Lambda_\lambda^\mu = (\Lambda^{-1})^\mu_\lambda A_a^\lambda(\Lambda x + a, \Lambda l).$$

- ★ Locality (causality): $[A_a^\mu(x, l), A_a^\lambda(x', l')] = 0$ when the strings $\{x + tl\}$ and $\{x' + t'l'\}$ are causally disjoint.

The first three properties are nearly obvious. The last one is subtler. It follows from (an easy variant of) the powerful argument in [16], based on modular localization theory, spelled out in Appendix D.

Explicitly, in terms of (2.1), one finds that:

$$A_a^\mu(x, l) = \sum_r \int d\mu(p) [e^{i(p x)} u_r^\mu(p, l) \alpha_{r,a}^\dagger(p) + e^{-i(p x)} u_r^\mu(p, l)^* \alpha_{r,a}(p)],$$

with $u_r^\mu(p, l) := \int_0^\infty dt e^{it(pl)} i(p^\mu e_r^\lambda(p) - p^\lambda e_r^\mu(p)) l_\lambda = e_r^\mu(p) - p^\mu \frac{(e_r(p) l)}{(pl)}. \quad (2.3)$

In the massless case, $(pl)^{-1}$ is understood to mean $((pl) + i0)^{-1}$. In keeping with the nomenclature of [11, 12], the quantities $u_r^\mu(p, l)$, $u_r^\mu(p, l)^*$, and similar ones for stringlike or pointlike fields, are here called *intertwiners*.

For the massive particles W^\pm and Z , it proves useful to consider the spinless string-local *escort* fields:

$$\phi_b(x, l) := \sum_r \int d\mu(p) \left[e^{i(p x)} \frac{i(e_r(p) l)}{(pl)} \alpha_{r,b}^\dagger(p) + e^{-i(p x)} \frac{-i(e_r(p) l)^*}{(pl)} \alpha_{r,b}(p) \right]. \quad (2.4)$$

We remark that

$$A_b^\mu(x, l) - \partial^\mu \phi_b(x, l) =: A_b^{p,\mu}(x), \quad (2.5)$$

defines pointlike *Proca* fields, so that $dA_b^p = F_b$. All these fields live on the same Fock spaces as the F_b and have the same mass. Moreover:

$$\phi_b(x, l) = \int_0^\infty A_b^{p,\lambda}(x + sl) l_\lambda ds.$$

¹Spacelike strings were employed in this literature. It is nevertheless better to deal here with lightlike strings. Our argument works either way [15].

²Here and later, $(\partial A) = \partial_\mu A^\mu$ denotes a divergence.

Note the relations $(l \partial \phi_b) = -(l A_b^p)$ and

$$\partial_\mu A_b^\mu(x, l) + m_b^2 \phi_b(x, l) = 0.$$

The last relation follows directly from (2.3) and (2.4), since $(p e_r(p)) = 0$.

Let now $d_l := \sum_\sigma dl^\sigma (\partial / \partial l^\sigma)$ denote the differential with respect to the string coordinate. We may introduce the (form-valued in the string variable) field:

$$\begin{aligned} d_l \phi_b(x, l) = w_b(x, l) := \sum_r \int d\mu(p) \left[e^{i(px)} \left(\frac{i e_{r,\sigma}(p)}{(pl)} - \frac{i p_\sigma (e_r(p) l)}{(pl)^2} \right) \alpha_{r,b}^\dagger(p) \right. \\ \left. + e^{-i(px)} \left(\frac{i e_{r,\sigma}(p)}{(pl)} - \frac{i p_\sigma (e_r(p) l)}{(pl)^2} \right)^* \alpha_{r,b}(p) \right] dl^\sigma; \end{aligned} \quad (2.6)$$

and one obtains

$$\begin{aligned} \partial_\mu w_b = - \sum_r \int d\mu(p) \left[e^{i(px)} \left(\frac{p_\mu e_{r,\sigma}(p)}{(pl)} - \frac{p_\mu p_\sigma (e_r(p) l)}{(pl)^2} \right) \alpha_{r,b}^\dagger(p) \right. \\ \left. + e^{-i(px)} \left(\frac{p_\mu e_{r,\sigma}(p)}{(pl)} - \frac{p_\mu p_\sigma (e_r(p) l)}{(pl)^2} \right)^* \alpha_{r,b}(p) \right] dl^\sigma = d_l A_b^\mu; \end{aligned}$$

as well as $d_l w_b := d_l^2 \phi_b = 0$. In the case that A_a^μ describes a massless field, we just take the second equality in (2.6) as *definition* of w_a – note that $((pl) \pm i0)^{-2}$ both exist as distributions – and $d_l A_\gamma^\mu = \partial^\mu w_\gamma$ still holds.³

We hasten now to exhibit a family of (Wightman) two-point functions for our fields, of the general form

$$\langle\langle \varphi(x, l) \psi(x', l) \rangle\rangle = \frac{1}{(2\pi)^3} \int d^4 p e^{-i(p(x-x'))} \delta_+(p^2 - m^2) M^{\varphi\psi}(p, l);$$

where any of the two fields φ, ψ , belong to the collection

$$\{ F_a^{\mu\nu}(x), A_a^\mu(x, l), \phi_b(x, l), \partial^\mu \phi_b(x, l), w_a(x, l), \partial^\mu w_a(x, l) \}$$

with a running over $(1, 2, 3, 4)$ and b over $(1, 2, 3)$. We shall suppress the subindex notation a, b in the rest of this section.

The respective $M^{\varphi\psi}$ are computed from the definitions of the fields. It is enough to note that:

$$M_{\alpha\beta}^{\varphi\psi} := \sum_r u_{r,\alpha}^{(\varphi)}(p, l)^* u_{r,\beta}^{(\psi)}(p, l),$$

in terms of intertwiners $u^{(\varphi)}, u^{(\psi)}$ already given. We get, to begin with,

$$M_{\mu\nu}^{AA} = -g_{\mu\nu} + \frac{p_\mu l_\nu + p_\nu l_\mu}{(pl)}. \quad (2.7a)$$

³Strictly speaking, w_γ exists in an inequivalent representation, due to expected infrared problems: its two-point function can be smeared only with test functions with zero moment.

The noteworthy and truly valuable fact here is that this is of order 0 as $p^2 \uparrow \infty$, while the two-point function of a Proca field goes like p^2 . The formula is analogous to that which comes out of lightcone gauge-fixing [17]. However, the meaning is quite different; in particular, our formalism is fully covariant. On configuration space, therefore, $\langle\langle A(x, l) A(x', l) \rangle\rangle$ essentially scales like λ^{-2} under $x \mapsto \lambda x$, whereas $\langle\langle A^P(x) A^P(x') \rangle\rangle$ goes as λ^{-4} .

Let us fill up a little table of vacuum expectation values of field products [18]:

$$\begin{aligned}
M_{\mu\nu, \rho\sigma}^{FF} &= -(p_\mu p_\rho g_{\nu\sigma} - p_\nu p_\rho g_{\mu\sigma} - p_\mu p_\sigma g_{\nu\rho} + p_\nu p_\sigma g_{\mu\rho}), \\
M_{\mu\nu, \lambda}^{\partial A, A} &= i \left(p_\mu g_{\nu\lambda} - p_\mu \frac{p_\nu l_\lambda + p_\lambda l_\nu}{(pl)} \right), \\
M_{\mu\nu, \lambda}^{FA} &= i \left(p_\mu g_{\nu\lambda} - p_\nu g_{\mu\lambda} - \frac{p_\mu l_\nu - p_\nu l_\mu}{(pl)} p_\lambda \right), \\
M_{\mu\nu}^{F\phi} &= \frac{p_\nu l_\mu - p_\mu l_\nu}{(pl)}, \quad M_\mu^{A\phi} = -\frac{il_\mu}{(pl)}, \quad M_{\mu\nu}^{A, \partial\phi} = \frac{p_\nu l_\mu}{(pl)}, \\
M_\mu^{\phi\phi} &= \frac{1}{m^2}, \quad M_\mu^{\partial\phi, \phi} = -\frac{ip_\mu}{m^2},
\end{aligned} \tag{2.7b}$$

as well as

$$\begin{aligned}
M_\mu^{Aw} &= \frac{i}{(pl)} M_{\mu\sigma}^{AA} dl^\sigma = i \left(\frac{-g_{\mu\sigma}}{(pl)} + \frac{p_\sigma l_\mu}{(pl)^2} \right) dl^\sigma, \quad M^{w\phi} = 0, \\
M^{ww} &= \frac{1}{(pl)^2} M_{\sigma\tau}^{AA} dl^\sigma \wedge dl^\tau = -\frac{g_{\sigma\tau}}{(pl)^2} dl^\sigma \wedge dl^\tau,
\end{aligned} \tag{2.7c}$$

using the relation $l_\sigma dl^\sigma = 0$. It is clear that massless bosons do not bear escort quantum fields.

The construction of SLF for spin 2 or helicity ± 2 proceeds in the same way, from the equivalent object to the Faraday tensor F , the linearized Riemann tensor R for spin or helicity 2, towards the string-local replacement for the pointlike (symmetric rank 2 tensor) “potential”. Note that physical scalar fields are not stringy.⁴

Now we may summarize some of the advantages offered by SL fields over the humdrum sort, at the price of an extra variable. The stringy fields evade the theorem that it is impossible to construct on Hilbert space a *vector* field for massless particles of helicity ± 1 [7, Sect. 5.9]; our A_a^μ are true vector fields. For this reason, the concept of gauge fades into the background. The improved ultraviolet behaviour for spin and helicity $> \frac{1}{2}$ permits one to get rid of the ghosts. Arguably, it is inherited from the amazingly good behaviour of the field strengths themselves, beyond naïve power counting, independent of spin, uncovered recently [20, 21].

Renormalization of SLF models takes place without calling upon ghost fields, BRS invariance and the like: *one need not ever surrender positivity* of the energy and of the state spaces for the physical particles – see the following sections.

⁴Nor are free Dirac fields; SLF for half-integer spin greater than $\frac{1}{2}$ or integer spin greater than 2 are discussed elsewhere [18, 19].

Not least, democracy is reestablished among the Wigner unirreps, since the (boson and fermion) Wigner unbounded-helicity particles [6], with Casimirs $P^2 = 0$, $W^2 < 0$, that have no corresponding pointlike fields [22, 23], become admitted into the realm of quantum field theory through SL fields [11, 12].

3 A matter of principle

A lesson of gauge field theory is that couplings of quantum fields should fall out from a simple underlying principle. The natural and essential hypothesis of interacting SLF theory is simple enough: physical observables and quantities closely related to them, particularly the \mathbb{S} -matrix, cannot depend on the string coordinates. This is the *string-independence* principle: colloquially, the string “ought not to be seen”.

Our framework for electroweak theory is outlined next. This both exemplifies the principle and contributes to the core of this paper.

- ★ The couplings between interaction carriers and matter currents in a theory with massive or massless vector bosons $A_{a\mu}$ are of the general form

$$g(b^a(A_a J_V) + \tilde{b}^a(A_a J_A)) := g(b^a A_{a\mu} J_V^\mu + \tilde{b}^a A_{a\mu} J_A^\mu); \quad (3.1)$$

where $J_V^\mu = \bar{\psi} \gamma^\mu \psi$, $J_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$,

with electric charge conserved in the interaction vertices.

- ★ The A_a^μ of above are given as string-local quantum fields. This is the key assumption.
- ★ The ψ in (3.1) are ordinary fermion fields – we should not assume chiral fermions *ab initio*, and we do not.
- ★ The b^a and \tilde{b}^a coefficients in (3.1) are to be determined from string independence.

Let T_1 denote a first-order vertex coupling in general. For the physics of the model described by T_1 to be string-independent, we must require that a vector field $Q_1^\mu(x, l)$ exist such that

$$d_l T_1 = (\partial Q_1) \equiv \partial_\mu Q_1^\mu, \quad (3.2)$$

so that, regarding the \mathbb{S} -matrix as the “adiabatic limit” of Bogoliubov’s functional \mathbb{S} -matrix [24] – as discussed in the next section – on applying integration by parts, the contribution from the divergence vanishes. Note moreover that (perturbative) string independence must hold at every order in the couplings, and must survive renormalization.

It turns out that the principle holds great power both as a heuristic device and a justification tool, dictating *symmetry* (of the Abelian and non-Abelian kind) from interaction⁵ down to almost every nut and bolt. A complete account of electroweak theory would start by

⁵Thus reversing Yang’s *dictum*, restated in the famous terminological discussion on gauge interactions between Dirac, Ferrara, Kleinert, Martin, Wigner, Yang himself and Zichichi [25].

showing that, when the string-independence principle is applied to the relevant set of SLF, with their known masses and charges – call them $A_{\pm}(x, l)$, $A_Z(x, l)$, $A_{\gamma}(x, l)$ – plus one *physical* Higgs particle $\phi_4(x)$, replacing the standard pointlike fields,⁶ one recovers precisely the phenomenological couplings of flavourdynamics in the Standard Model (SM), with massive bosons mediating the weak interactions, and the $U(2)$ structure constants, as for instance in [27] or [28, Ch. 1]. (We cannot quite say that we recover *the* Standard Model picture after spontaneous symmetry breaking has allegedly taken place, since our boson fields are different, and our rule set cares little for Lagrangians. But the coincidence of the couplings ought to be evident – see the discussion at the end of Section 7.) Such a derivation, spelled out in future papers (see [29, 30]), requires one to examine time-ordered products corresponding to graphs involving boson particles up to third order in the couplings. Here we just devote the following short subsection to display its flavour, and foremost the results we need, to garner our derivation for chirality of weak interactions.

It is time to declare that a valid argument for chirality, with the same outcome as ours, can be made, and has indeed been made before, within the conventional frame of Kugo–Ojima asymptotic fields [31–33]. Apparently this proof was scarcely heeded, for reasons mostly beyond our comprehension. It is, certainly, couched in the language of (the causal version of) gauge theory, keeping its ungainly retinue of unphysical fields. The string-local paradigm provides a cleaner, “native” form. Still, it was a good case, and we are keen to employ new tools to reclaim it.

3.1 On the string-local boson sector

Apart from the higgs particle sector, a string-local theory of interacting bosons at first order in the coupling constant g must be of the form [30]:

$$T_1^B(x, l) = g \sum_{a,b,c} f_{abc} F_a(x) A_b(x, l) A_c(x, l) \quad (3.3)$$

$$+ g \sum'_{a,b,c} f_{abc} (m_a^2 - m_b^2 - m_c^2) (A_a(x, l) A_b(x, l) \phi_c(x, l) - A_a(x, l) \partial \phi_b(x, l) \phi_c(x, l)),$$

where the restricted sum \sum' runs over massive fields only. Notice that the escort fields hold a somewhat analogous place to Stückelberg fields. Here the f_{abc} denote the (completely skewsymmetric) structure constants of the (reductive) symmetry group of the model; the mass of the vector boson A_a is denoted m_a , and complete contraction of Lorentz indices is understood.

Now it is easy to check that the 1-form $d_l T_1^B$, measuring the dependence on the string variable of the vertices in (3.3), is a divergence: $d_l T_1^B = (\partial Q_1^B)$, where

$$Q_{1\mu}^B(x, l) := 2g \sum_{a,b,c} f_{abc} (F_a A_c)_\mu w_b + g \sum'_{a,b,c} f_{abc} (m_a^2 + m_c^2 - m_b^2) (A_{a\mu} - \partial_\mu \phi_a) \phi_c w_b. \quad (3.4)$$

⁶Following Okun [26], and for obvious grammatical reasons, henceforth we refer to a (physical) Higgs boson as a higgs, with a lower-case h.

We shall need Q_1^B to prove chirality of the couplings to the fermion sector.

At once we adapt our notation to the one used in the SM. This model has three masses m_1, m_2, m_3 different from zero and one $m_4 = m_\gamma = 0$. By studying a key vertex between ϕ_4 – assumed unique – and the stringy boson fields, and discarding a couple of trivial cases, one finds that $m_1 = m_2 = m_W \leq m_3 = m_Z$; thus, *defining* the Weinberg angle by

$$\frac{m_W}{m_Z} =: \cos \Theta,$$

one is further able to derive:

$$f_{123} = \frac{1}{2} \cos \Theta, \quad f_{124} = \frac{1}{2} \sin \Theta, \quad f_{134} = f_{234} = 0.$$

All other f_{abc} follow from complete skewsymmetry. They are seen to be the structure constants of (the Lie algebra of) the $U(2)$ group in a given basis – determined by the physical particle fields.

We shall use the standard notations

$$W_\pm \equiv \frac{1}{\sqrt{2}}(W_1 \mp iW_2) := \frac{1}{\sqrt{2}}(A_1 \mp iA_2), \quad Z := A_3, \quad A := A_4$$

and similarly for ϕ_\pm, w_\pm, ϕ_Z and w_Z .

With this in hand, we focus on (3.4), keeping in mind that, although an escort field does not exist for the photon, the field w_4 exists at the same title as w_1, w_2 and w_Z . The first summand in (3.4) yields:

$$\begin{aligned} & 2g \sum f_{abc} (\partial_\mu A_{a\lambda} - \partial_\lambda A_{a\mu}) A_c^\lambda w_b \\ &= ig \sin \Theta [(\partial_\mu A_\lambda - \partial_\lambda A_\mu)(w_- W_+^\lambda - w_+ W_-^\lambda) + (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu})(w_+ A^\lambda - w_4 W_+^\lambda) \\ & \quad + (\partial_\mu W_{+\lambda} - \partial_\lambda W_{+\mu})(w_4 W_-^\lambda - w_- A^\lambda)] \\ &+ ig \cos \Theta [(\partial_\mu Z_\lambda - \partial_\lambda Z_\mu)(w_- W_+^\lambda - w_+ W_-^\lambda) + (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu})(w_+ Z^\lambda - w_Z W_+^\lambda) \\ & \quad + (\partial_\mu W_{+\lambda} - \partial_\lambda W_{+\mu})(w_Z W_-^\lambda - w_- Z^\lambda)]. \end{aligned} \quad (3.5)$$

Our $Q_{1\mu}^B$ above is not complete, since bosonic couplings involving the higgs sector have not been included. For our purposes here we employ only:

$$\frac{g}{2 \cos \Theta} m_Z (\phi_4 (\partial_\mu \phi_Z - Z_\mu) - \partial_\mu \phi_4 \phi_Z) w_Z. \quad (3.6)$$

Terms of this type are suggested by the last group of summands in (3.4).⁷ We shall see later that the higgs' couplings, as derived from the string-independence principle, play a pivotal role in our problem. Also, it is worthwhile to mention that “the SM accounts for, but does not explain, electroweak symmetry breaking” [34]. This alludes to the “negative squared mass” in the higgs' potential. Here again, SLF theory goes one better: that potential can be derived from string independence. We refer to [29] in this respect.

⁷The notation is not meant to purport the higgs as a rogue escort!

4 Perturbation theory for string-local fields

New theories demand care with the mathematics. For this reason, like the authors of [31], we borrow the Stückelberg–Bogoliubov–Epstein–Glaser (SBEG) renormalization-without-regularization formalism for perturbation theory, both most rigorous and flexible. For want of space, we only outline here what we need. For details, the original works [24, 35] remain the best.

The method involves the construction of a scattering operator $\mathbb{S}[g; l]$ functionally dependent on a (multiplet of) smooth external fields $g(x)$, which mathematically are test functions. The procedure is natural in view of locality; the functional scattering operator acts on the Fock spaces corresponding to local free fields, of the pointlike or stringlike variety, for a prescribed set of free particles. It is submitted to the following conditions.

★ Covariance:

$$U(a, \Lambda) \mathbb{S}[g; l] U^\dagger(a, \Lambda) = \mathbb{S}[(a, \Lambda)g; \Lambda l],$$

where $(a, \Lambda)g(x) = g(\Lambda^{-1}(x - a))$.

★ Unitarity:

$$\mathbb{S}^{-1}[g; l] = \mathbb{S}^\dagger[g; l].$$

★ Causality. Let V^+ , V^- denote the future and past solid light cones. Then

$$\mathbb{S}[g_1 + g_2; l] = \mathbb{S}[g_1; l] \mathbb{S}[g_2; l] \quad (4.1)$$

when $\text{supp } g_2 + \mathbb{R}^+ l \cap \text{supp } g_1 + V^+ = \emptyset$, or equivalently $\text{supp } g_1 \cap (\text{supp } g_2 + V^- + \mathbb{R}^+ l) = \emptyset$.

In practice one looks for $\mathbb{S}[g; l]$ as a power series in g , of the form

$$\mathbb{S}[g; l] = 1 + \sum_{k=1}^{\infty} \frac{i^k}{k!} \int_{\mathbb{M}_4^k} T_k(x_1, \dots, x_k, l) g(x_1) \cdots g(x_k) dx_1 \cdots dx_k. \quad (4.2)$$

Only the first-order term T_1 is postulated. This will be a Wick polynomial in the free fields. In many models it has the look of an interaction Lagrangian. It however should be kept in mind that the building blocks in the procedure are quantum fields; ditto, our starting point is Wigner’s theory of quantum Poincaré modules [6] and corresponding field strength representations of the Lorentz group, rather than a classical Lagrangian that one should attempt to “quantize”. Interacting fields can be defined and proved to be causal and to fulfil Yang–Feldman equations, by adding a field source term in the definition of $\mathbb{S}[g; l]$; but we need not go into that in this paper.

In consonance with (4.1), the $T_k(x_1, \dots, x_k, l)$ for $k \geq 2$ are *time-ordered products*, which need to be constructed. Naïvely they correspond to products of distributions, ill-defined in general. By locality, the causal factorization

$$T_2(x, x', l) = T_1(x, l) T_1(x', l) \quad \text{or} \quad T_1(x', l') T_1(x, l),$$

according as $\{x + tl\}$ is later or earlier than $\{x' + tl\}$, fixes T_2 on a large region of $\mathbb{M}_4^k \times S^2$. Indeed, assuming $l^0 > 0$, a string $\{x + tl\}$ lies to the future of another string $\{x' + t'l\}$ if and only if $((x - x')l) \geq 0$ and the intersection of the strings is empty. That is, x lies to the future of, or on, the hyperplane $x' + l^\perp$, but not on the full line $x' + \mathbb{R}l$ [15]. Consequently, the strings cannot be ordered if and only if x lies on the string $\{x' + t'l\}$ or vice versa; i.e., if and only if $x - x'$ is lightlike and parallel to l . The set of such (x, l) , hereinafter called D , is of measure zero in $\mathbb{M}_4^k \times S^2$.

The extension of such products to the whole of $\mathbb{M}_4^k \times S^2$, mainly by upholding string independence, is our SBEG renormalization problem in a nutshell.

Existence of the adiabatic limit is the property that the T_k be integrable distributions, in the sense of Schwartz [36]. In that limit, as g goes into a constant, the covariant $\mathbb{S}[g; l]$ is expected to approach the invariant physical scattering matrix \mathbb{S} , so that in particular

$$U(a, \Lambda) \mathbb{S} U^\dagger(a, \Lambda) = \mathbb{S},$$

all dependence on the string disappearing.

Already the condition that $d_l T_1$ be a divergence severely restricts the interaction vertices in T_1 ; we proceed to throw light on the fermion sector by using it in the next section. Further along, all the time-ordered products T_k in the functional \mathbb{S} -matrix are to be determined from string independence. Our argument will involve their proper definition. (By the way, the $g^2 AAAAA$ terms and thus the classical geometrical gauge approach are recovered in our formalism from string independence at the level of T_2 .)

5 First-order constraints

The proof develops in two stages. In the first, we need not invoke the Q_1 -vector of the boson sector. For the coupling of the fermion sector of the SM with the vector bosons, in view of (3.1), and omitting the notation $:-:$ for the Wick products, we make the fairly general trilinear Ansatz:

$$\begin{aligned} T_1^F(x, l) := & g(b_1 W_{-\mu} \bar{e} \gamma^\mu v + \tilde{b}_1 W_{-\mu} \bar{e} \gamma^\mu \gamma^5 v + b_2 W_{+\mu} \bar{\nu} \gamma^\mu e + \tilde{b}_2 W_{+\mu} \bar{\nu} \gamma^\mu \gamma^5 e \\ & + b_3 Z_\mu \bar{e} \gamma^\mu e + \tilde{b}_3 Z_\mu \bar{e} \gamma^\mu \gamma^5 e + b_4 Z_\mu \bar{\nu} \gamma^\mu v + \tilde{b}_4 Z_\mu \bar{\nu} \gamma^\mu \gamma^5 v \\ & + b_5 A_\mu \bar{e} \gamma^\mu e + \tilde{b}_5 A_\mu \bar{e} \gamma^\mu \gamma^5 e + b_6 A_\mu \bar{\nu} \gamma^\mu v + \tilde{b}_6 A_\mu \bar{\nu} \gamma^\mu \gamma^5 v \\ & + c_1 \phi_- \bar{e} v + \tilde{c}_1 \phi_- \bar{e} \gamma^5 v + c_2 \phi_+ \bar{\nu} e + \tilde{c}_2 \phi_+ \bar{\nu} \gamma^5 e \\ & + c_3 \phi_Z \bar{e} e + \tilde{c}_3 \phi_Z \bar{e} \gamma^5 e + c_4 \phi_Z \bar{\nu} v + \tilde{c}_4 \phi_Z \bar{\nu} \gamma^5 v \\ & + c_0 \phi_4 \bar{e} e + \tilde{c}_0 \phi_4 \bar{e} \gamma^5 e + c_5 \phi_4 \bar{\nu} v + \tilde{c}_5 \phi_4 \bar{\nu} \gamma^5 v). \end{aligned} \quad (5.1)$$

All the boson fields here are string-local, except for the pointlike higgs field ϕ_4 . Here e stands for an electron, muon or τ -lepton pointlike field or for a (suitable combination of) quark fields d, s, b ; and v for the neutrinos or for the quarks u, c, t . Charge is conserved in each term. Nonvanishing fermion masses are assumed throughout.

It will be enough here to consider just one generation of leptons: bringing up the full structure of the fermion multiplets only complicates the proof's notation in a way immaterial to the purpose. Then the photon should not couple to neutrinos, which are uncharged, and we drop the corresponding terms, with coefficients b_6, \tilde{b}_6 , right away.

Unitarity of the S -matrix, in the light of (4.2), dictates that T_1 be hermitian. Thus, for instance, $b_2 = b_1^*$ and $\tilde{b}_2 = \tilde{b}_1^*$; and we may choose phases so that both b_1 and \tilde{b}_1 are real. Moreover, b_3, b_4, b_5, b_6 and $\tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6$ are all real; $c_2 = c_1^*$ and $\tilde{c}_2 = -\tilde{c}_1^*$; c_3, c_4, c_0, c_5 are real whereas $\tilde{c}_3, \tilde{c}_4, \tilde{c}_0, \tilde{c}_5$ are imaginary.

As indicated in Section 3, the ψ -fields e and ν are ordinary pointlike fermion fields. Let us use the Dirac equation to handle them; we could employ Weyl equations as well. The important feature is that the SBEG procedure is thoroughly an on-shell construction:

$$\overrightarrow{\not{D}} \psi = -im_\psi \psi, \quad \overleftarrow{\not{D}} \bar{\psi} = im_\psi \bar{\psi}. \quad (5.2)$$

String-independence at this order demands that there be a $Q_\mu^F(x, l)$ such that

$$d_l T_1^F(x, l) = \partial^\mu Q_\mu^F(x, l).$$

The left-hand side of this formula with the Ansatz (5.1) is trivially expressed with the help of the form-valued fields defined in (2.6):

$$\begin{aligned} d_l T_1^F(x, l) := & g(b_1 \partial_\mu w_- \bar{e} \gamma^\mu \nu + \tilde{b}_1 \partial_\mu w_- \bar{e} \gamma^\mu \gamma^5 \nu + b_1 \partial_\mu w_+ \bar{\nu} \gamma^\mu e + \tilde{b}_1 \partial_\mu w_+ \bar{\nu} \gamma^\mu \gamma^5 e \\ & + b_3 \partial_\mu w_Z \bar{e} \gamma^\mu e + \tilde{b}_3 \partial_\mu w_Z \bar{e} \gamma^\mu \gamma^5 e + b_4 \partial_\mu w_Z \bar{\nu} \gamma^\mu \nu + \tilde{b}_4 \partial_\mu w_Z \bar{\nu} \gamma^\mu \gamma^5 \nu \\ & + b_5 \partial_\mu w_4 \bar{e} \gamma^\mu e + \tilde{b}_5 \partial_\mu w_4 \bar{e} \gamma^\mu \gamma^5 e \\ & + c_1 w_- \bar{e} \nu + \tilde{c}_1 w_- \bar{e} \gamma^5 \nu + c_2 w_+ \bar{\nu} e + \tilde{c}_2 w_+ \bar{\nu} \gamma^5 e \\ & + c_3 w_Z \bar{e} e + \tilde{c}_3 w_Z \bar{e} \gamma^5 e + c_4 w_Z \bar{\nu} \nu + \tilde{c}_4 w_Z \bar{\nu} \gamma^5 \nu). \end{aligned}$$

Proposition 1. *There is a $Q_1^{F\mu}$ such that $d_l T_1^F(x, l) = \partial_\mu Q_1^{F\mu}(x, l)$, which is of the form*

$$\begin{aligned} Q_1^{F\mu} := & g(b_1 w_- \bar{e} \gamma^\mu \nu + \tilde{b}_1 w_- \bar{e} \gamma^\mu \gamma^5 \nu + b_1 w_+ \bar{\nu} \gamma^\mu e + \tilde{b}_1 w_+ \bar{\nu} \gamma^\mu \gamma^5 e \\ & + b_3 w_Z \bar{e} \gamma^\mu e + \tilde{b}_3 w_Z \bar{e} \gamma^\mu \gamma^5 e + b_4 w_Z \bar{\nu} \gamma^\mu \nu + \tilde{b}_4 w_Z \bar{\nu} \gamma^\mu \gamma^5 \nu + b_5 w_4 \bar{e} \gamma^\mu e), \end{aligned} \quad (5.3)$$

if and only if

$$\begin{aligned} c_1 &= i(m_e - m_\nu) b_1, & c_3 &= 0, \\ c_2 &= i(m_\nu - m_e) b_1, & c_4 &= 0, \\ \tilde{c}_1 &= i(m_e + m_\nu) \tilde{b}_1, & \tilde{c}_3 &= 2im_e \tilde{b}_3, \\ \tilde{c}_2 &= i(m_\nu + m_e) \tilde{b}_1, & \tilde{c}_4 &= 2im_\nu \tilde{b}_4, \quad \text{and} \quad \tilde{b}_5 = 0. \end{aligned} \quad (5.4)$$

Notice that there are no restrictions at this stage on the set $\{c_0, \tilde{c}_0, c_5, \tilde{c}_5\}$, since the corresponding vertices are pointlike.

Proof. By direct computation, using the Dirac equations and $\gamma^5 \gamma^\mu = -\gamma^\mu \gamma^5$, (5.3) gives:

$$\begin{aligned} \partial_\mu Q_1^{F\mu} = & g(b_1 \partial_\mu w_- \bar{e} \gamma^\mu v + \tilde{b}_1 \partial_\mu w_- \bar{e} \gamma^\mu \gamma^5 v + b_1 \partial_\mu w_+ \bar{v} \gamma^\mu e + \tilde{b}_1 \partial_\mu w_+ \bar{v} \gamma^\mu \gamma^5 e \\ & + b_3 \partial_\mu w_Z \bar{e} \gamma^\mu e + \tilde{b}_3 \partial_\mu w_Z \bar{e} \gamma^\mu \gamma^5 e + b_4 \partial_\mu w_Z \bar{v} \gamma^\mu v + \tilde{b}_4 \partial_\mu w_Z \bar{v} \gamma^\mu \gamma^5 v \\ & + i(m_e - m_v) b_1 w_- \bar{e} v + i(m_e + m_v) \tilde{b}_1 w_- \bar{e} \gamma^5 v + i(m_v - m_e) b_1 w_+ \bar{v} e \\ & + i(m_v + m_e) \tilde{b}_1 w_+ \bar{v} \gamma^5 e + 2im_e \tilde{b}_3 w_Z \bar{e} \gamma^5 e + 2im_v \tilde{b}_4 w_Z \bar{v} \gamma^5 v). \end{aligned}$$

The coefficient \tilde{b}_5 of the axial term for the photon must vanish since, being massless, it does not possess a scalar partner, and the axial current is not conserved. \square

Notice also that the argument for $\tilde{b}_5 = 0$ would have failed if the electron were massless. Whereas the axial terms for massive vector bosons in the original Ansatz have survived. They will keep surviving, as we shall see.

It is pertinent to substitute expressions (5.4) into (5.1), which we do now for convenience later on:

$$\begin{aligned} T_1^F(x, l) = & g(b_1 W_{-\mu} \bar{e} \gamma^\mu v + \tilde{b}_1 W_{-\mu} \bar{e} \gamma^\mu \gamma^5 v + b_1 W_{+\mu} \bar{v} \gamma^\mu e + \tilde{b}_1 W_{+\mu} \bar{v} \gamma^\mu \gamma^5 e \\ & + b_3 Z_\mu \bar{e} \gamma^\mu e + \tilde{b}_3 Z_\mu \bar{e} \gamma^\mu \gamma^5 e + b_4 Z_\mu \bar{v} \gamma^\mu v + \tilde{b}_4 Z_\mu \bar{v} \gamma^\mu \gamma^5 v + b_5 A_\mu \bar{e} \gamma^\mu e \\ & + i(m_e - m_v) b_1 \phi_- \bar{e} v + i(m_e + m_v) \tilde{b}_1 \phi_- \bar{e} \gamma^5 v - i(m_e - m_v) b_1 \phi_+ \bar{v} e \\ & + i(m_e + m_v) \tilde{b}_1 \phi_+ \bar{v} \gamma^5 e + 2im_e \tilde{b}_3 \phi_Z \bar{e} \gamma^5 e + 2im_v \tilde{b}_4 \phi_Z \bar{v} \gamma^5 v \\ & + c_0 \phi_4 \bar{e} e + \tilde{c}_0 \phi_4 \bar{e} \gamma^5 e + c_5 \phi_4 \bar{v} v + \tilde{c}_5 \phi_4 \bar{v} \gamma^5 v). \end{aligned} \quad (5.5)$$

6 Extracting T_2

We consider now a family of *Feynman* two-point functions for our fields, of the general form

$$\langle\langle T_0 \varphi(x, l) \chi(x', l) \rangle\rangle = \frac{i}{(2\pi)^4} \int d^4 p \frac{e^{-i(p(x-x'))}}{p^2 - m^2 + i0} M^{\varphi\chi}(p, l); \quad (6.1)$$

where the collection of fields φ, χ runs also over ϕ_4 . Note that, by definition,

$$D_F(x) := \frac{1}{(2\pi)^4} \int d^4 p \frac{e^{-i(px)}}{p^2 - m^2 + i0}, \quad \text{so that} \quad (\square + m^2) D_F(x) = -\delta(x).$$

It is easy to see that in general:

$$d_l \langle\langle T_0 \varphi \chi' \rangle\rangle = \langle\langle T_0 d_l \varphi \chi' \rangle\rangle + \langle\langle T_0 \varphi d_l \chi' \rangle\rangle. \quad (6.2)$$

We have denoted $\chi' \equiv \chi(x', l)$. On (eventually) renormalizing the time-ordered product T_0 to a more suitable one T , we expect that property (6.2) be likewise applicable to T .

Next we seek to enforce string-independence of time-ordered products *at second order* in the coupling constant. The construction will require dealing with derived fields. Since the

factorization $T_2(x, x', l) = T_1(x, l)T_1(x', l)$ or $T_1(x', l)T_1(x, l)$ holds for almost all $(x - x', l)$, we require that the relation

$$d_l T_0[T_1(x, l)T_1(x', l)] = \partial_\mu T_0[Q_1^\mu(x, l)T_1(x', l)] + \partial'_\mu T_0[T_1(x, l)Q_1^\mu(x', l)] \quad (6.3)$$

be valid everywhere; here both T_1 and Q_1 will contain (purely) bosonic and fermionic contributions. It will be enough to examine $d_l T_0[T_1 T_1']$ in (6.3) with d_l acting only on T_1 , since both summands in the equation give identical contributions.

As advertised, we shall only need to examine *tree graphs* in T_2 to determine the coefficients in (5.1). Employing formal derivation within the Wick product couplings U, V' , those graphs are defined by

$$T_0[UV']_{\text{tree}} = \sum_{\varphi, \chi'} \frac{\partial U}{\partial \varphi} \langle T_0 \varphi \chi' \rangle \frac{\partial V'}{\partial \chi'}. \quad (6.4)$$

With the help of our previous assumptions, and just writing Q for Q_1 , we reckon that:

$$\begin{aligned} & \left[\sum_{\varphi, \chi'} d_l \left(\frac{\partial T_1}{\partial \varphi} \langle T_0 \varphi \chi' \rangle \right) - \sum_{\psi, \chi'} \partial_\mu \left(\frac{\partial Q^\mu}{\partial \psi} \langle T_0 \psi \chi' \rangle \right) \right] \frac{\partial T_1'}{\partial \chi'} \\ &= \left[\sum_{\varphi, \chi'} d_l \frac{\partial T_1}{\partial \varphi} \langle T_0 \varphi \chi' \rangle + \frac{\partial T_1}{\partial \varphi} \langle T_0 d_l \varphi \chi' \rangle \right. \\ & \quad \left. - \sum_{\psi, \chi'} \left(\partial_\mu \frac{\partial Q^\mu}{\partial \psi} \langle T_0 \psi \chi' \rangle + \frac{\partial Q^\mu}{\partial \psi} \langle T_0 \partial_\mu \psi \chi' \rangle \right) \right] \frac{\partial T_1'}{\partial \chi'} \\ &+ \left[\sum_{\varphi, \chi'} \frac{\partial T_1}{\partial \varphi} (d_l \langle T_0 \varphi \chi' \rangle - \langle T_0 d_l \varphi \chi' \rangle) - \sum_{\psi, \chi'} \frac{\partial Q^\mu}{\partial \psi} (\partial_\mu \langle T_0 \psi \chi' \rangle - \langle T_0 \partial_\mu \psi \chi' \rangle) \right] \frac{\partial T_1'}{\partial \chi'}. \end{aligned} \quad (6.5)$$

The first bracketed term on the right hand side reduces to the tree-graph contribution

$$T_0[(d_l T_1) \chi']_{\text{tree}} - T_0[(\partial_\mu Q^\mu) \chi']_{\text{tree}}, \quad (6.6)$$

which vanishes by construction. We refer to Appendix B for the proof of that equality. The following two terms cancel because of (6.2). Therefore, the entire expression (6.5) reduces to the last pair of summands:

$$O := \sum_{\psi, \chi'} \frac{\partial Q^\mu}{\partial \psi} (\langle T_0 \partial_\mu \psi \chi' \rangle - \partial_\mu \langle T_0 \psi \chi' \rangle) \frac{\partial T_1'}{\partial \chi'}. \quad (6.7)$$

We then seek to determine this last quantity (6.7), that we have called O for “(total) obstruction to string independence”. The argument developed so far, from Eqs. (6.2) to (6.7), contains time-ordered products $\langle T_0 \varphi \chi' \rangle$ given by (6.1). These may need to be replaced by renormalized time-ordered products $\langle T \varphi \chi' \rangle$. The latter are fixed, *a priori*, up to numerical distributions with support on the manifold D of codimension 3. Such distributions may be added only if the scaling degree of the original time-ordered product is at least 3. Vanishing of O , with T_0 replaced by T as necessary, will provide the correct couplings, and in the

occasion chirality of the interaction of the fermions with the massive intermediate vector bosons.

We now start to examine 2-point obstructions of type

$$\mathcal{O}(C, \psi') := \langle\langle T_0 \partial_\mu C^\mu \psi' \rangle\rangle - \partial_\mu \langle\langle T_0 C^\mu \psi' \rangle\rangle.$$

First of all, there are two that vanish:

$$\mathcal{O}(A, \phi') := \langle\langle T_0 \partial_\mu A^\mu \phi' \rangle\rangle - \partial_\mu \langle\langle T_0 A^\mu \phi' \rangle\rangle = 0, \quad (6.8a)$$

$$\mathcal{O}(\partial_\lambda A, \phi') := \langle\langle T_0 \partial_\mu \partial_\lambda A^\mu \phi' \rangle\rangle - \partial_\mu \langle\langle T_0 \partial_\lambda A^\mu \phi' \rangle\rangle = 0. \quad (6.8b)$$

Indeed, the left hand side of (6.8a) is $-m^2 \langle\langle T_0 \phi \phi' \rangle\rangle - \partial_\mu \langle\langle T_0 A^\mu \phi' \rangle\rangle$, which vanishes because

$$\partial_\mu \langle\langle T_0 A^\mu \phi' \rangle\rangle = \frac{-i}{(2\pi)^4} \int d^4 p \frac{e^{-i(p(x-x'))}}{p^2 - m^2 + i0} \equiv -i D^F(x - x') = -m^2 \langle\langle T_0 \phi \phi' \rangle\rangle,$$

in view of (2.7b). Thus (6.8a) holds; and a similar calculation yields (6.8b).

Next, we consider

$$\mathcal{O}(A, A'_\kappa) := \langle\langle T_0 \partial_\mu A^\mu A'_\kappa \rangle\rangle - \partial_\mu \langle\langle T_0 A^\mu A'_\kappa \rangle\rangle.$$

Using (2.7), we get

$$\mathcal{O}(A, A'_\kappa) = \frac{1}{(2\pi)^4} \int d^4 p \frac{e^{-i(p(x-x'))}}{p^2 - m^2 + i0} \frac{(m^2 - p^2) l_\kappa}{(pl)} = -\frac{l_\kappa}{(2\pi)^4} \int d^4 p \frac{e^{-i(p(x-x'))}}{(pl)}.$$

On bringing in the distributions

$$\frac{1}{(pl)} = -i \int_0^\infty ds e^{is(pl)} \quad \text{and} \quad \delta_l(x) := \int_0^\infty ds \delta(x - sl),$$

we may rewrite the obstruction as

$$\mathcal{O}(A, A'_\kappa) = \frac{il_\kappa}{(2\pi)^4} \int_0^\infty ds \int d^4 p e^{-i(p(x-x'-sl))} = il_\kappa \delta_l(x - x'). \quad (6.9)$$

The distribution δ_l is supported on D , and possesses the correct scaling degree.

We next determine

$$\begin{aligned} \mathcal{O}(\partial \phi, A'_\kappa) &:= \langle\langle T_0 \partial_\mu \partial^\mu \phi A'_\kappa \rangle\rangle - \partial_\mu \langle\langle T_0 \partial^\mu \phi A'_\kappa \rangle\rangle \\ &= -(\square + m^2) \langle\langle T_0 \phi A'_\kappa \rangle\rangle = -\frac{1}{(2\pi)^4} \int d^4 p e^{-i(p(x-x'))} \frac{l_\kappa}{(pl)} = il_\kappa \delta_l. \end{aligned}$$

Since \mathcal{O} is bilinear in its arguments, this yields a useful

$$\mathcal{O}(A - \partial \phi, A'_\kappa) = 0.$$

Likewise,

$$\mathcal{O}(\partial A_\lambda, \phi') := \langle\langle T_0 \partial_\mu \partial^\mu A_\lambda \phi' \rangle\rangle - \partial_\mu \langle\langle T_0 \partial^\mu A_\lambda \phi' \rangle\rangle = -(\square + m^2) \langle\langle T_0 A_\lambda \phi' \rangle\rangle = -il_\lambda \delta_l.$$

The scaling degrees of the expectation values computed up to now are lower than the codimension of D . On the other hand, we need to compute $\langle\langle T_0 \partial_\lambda A_\mu A'_\kappa \rangle\rangle$, whose scaling degree with respect to D equals the codimension of the latter, namely 3. Therefore, we have to admit the possibility of adding to $\langle\langle T_0 \partial_\lambda A_\mu A'_\kappa \rangle\rangle$ a numerical distribution with support on D and with the same scaling degree. Any such distribution is of the form $c_{\lambda\mu\kappa}(x) \delta_l$ for some well-behaved function $c_{\lambda\mu\kappa}$. Consequently, in principle we must introduce

$$\langle\langle T \partial_\lambda A_\mu A'_\kappa \rangle\rangle := \langle\langle T_0 \partial_\lambda A_\mu A'_\kappa \rangle\rangle + c_{\lambda\mu\kappa} \delta_l \quad (6.10)$$

for suitable coefficients $c_{\lambda\mu\kappa}$, as yet undetermined.

We are now ready to tackle the more challenging obstruction

$$\begin{aligned} \mathcal{O}(\partial_\lambda A, A'_\kappa) &:= \langle\langle T \partial_\mu \partial_\lambda A^\mu A'_\kappa \rangle\rangle - \partial_\mu \langle\langle T \partial_\lambda A^\mu A'_\kappa \rangle\rangle \\ &= \partial_\lambda (-m^2 \langle\langle T_0 \phi A'_\kappa \rangle\rangle - \partial^\mu \langle\langle T_0 A_\mu A'_\kappa \rangle\rangle) - \partial^\mu (c_{\lambda\mu\kappa} \delta_l) \\ &= il_\kappa \partial_\lambda \delta_l - \partial^\mu (c_{\lambda\mu\kappa} \delta_l). \end{aligned} \quad (6.11)$$

Next, we find, using (2.7a) and (6.9), that

$$\begin{aligned} \mathcal{O}(\partial A_\lambda, A'_\kappa) &:= \langle\langle T \partial_\mu \partial^\mu A_\lambda A'_\kappa \rangle\rangle - \partial_\mu \langle\langle T \partial^\mu A_\lambda A'_\kappa \rangle\rangle \\ &= -(\square + m^2) \langle\langle T_0 A_\lambda A'_\kappa \rangle\rangle - \partial^\mu (c_{\mu\lambda\kappa} \delta_l) \\ &= -ig_{\lambda\kappa} \delta + i(l_\lambda \partial_\kappa + l_\kappa \partial_\lambda) \delta_l - \partial^\mu (c_{\mu\lambda\kappa} \delta_l). \end{aligned} \quad (6.12)$$

On subtracting (6.11) from (6.12), we arrive at

$$\mathcal{O}(F_{\bullet\lambda}, A'_\kappa) \equiv \mathcal{O}(\partial_\lambda A - \partial A_\lambda, A'_\kappa) = -ig_{\lambda\kappa} \delta + il_\lambda \partial_\kappa \delta_l + \partial^\mu (c_{[\lambda\mu]\kappa} \delta_l),$$

with the skewsymmetrization $c_{[\lambda\mu]\kappa} \equiv c_{\lambda\mu\kappa} - c_{\mu\lambda\kappa}$.

Finally, we take note of

$$\begin{aligned} \mathcal{O}(\partial \phi_a, \phi'_a) &:= \langle\langle T_0 \partial_\mu \partial^\mu \phi_a \phi'_a \rangle\rangle - \partial_\mu \langle\langle T_0 \partial^\mu \phi_a \phi'_a \rangle\rangle = \frac{i}{m_a^2} \delta \quad \text{for } a = 1, 2, 3, \\ \mathcal{O}(\partial \phi_4, \phi'_4) &:= \langle\langle T_0 \partial_\mu \partial^\mu \phi_4 \phi'_4 \rangle\rangle - \partial_\mu \langle\langle T_0 \partial^\mu \phi_4 \phi'_4 \rangle\rangle = i \delta. \end{aligned}$$

To sum up: the obstructions of bosonic type are:

$$\begin{aligned} \mathcal{O}(A, \phi') &= 0, & \mathcal{O}(A, A'_\kappa) &= il_\kappa \delta_l, \\ \mathcal{O}(\partial_\lambda A, \phi') &= 0, & \mathcal{O}(\partial_\lambda A, A'_\kappa) &= il_\kappa \delta_l, \\ \mathcal{O}(A - \partial \phi, A'_\kappa) &= 0, & \mathcal{O}(\partial A_\lambda, \phi') &= -il_\lambda \delta_l, \\ \mathcal{O}(\partial \phi_a, \phi'_a) &= (i/m_a^2) \delta, & \mathcal{O}(\partial_\lambda A, A'_\kappa) &= il_\kappa \partial_\lambda \delta_l - \partial^\mu (c_{\lambda\mu\kappa} \delta_l), \\ \mathcal{O}(\partial \phi_4, \phi'_4) &= i \delta, & \mathcal{O}(\partial A_\lambda, A'_\kappa) &= -ig_{\lambda\kappa} \delta + i(l_\lambda \partial_\kappa + l_\kappa \partial_\lambda) \delta_l - \partial^\mu (c_{\mu\lambda\kappa} \delta_l), \\ & & \mathcal{O}(F_{\bullet\lambda}, A'_\kappa) &= -ig_{\lambda\kappa} \delta + il_\lambda \partial_\kappa \delta_l + \partial^\mu (c_{[\lambda\mu]\kappa} \delta_l). \end{aligned} \quad (6.13)$$

The fermionic obstructions, which do not involve stringlike fields, are much simpler. They are of two kinds, where ψ, ψ' denote two fermions of the same type:

$$\begin{aligned}\mathcal{O}(\gamma\psi, \bar{\psi}') &:= \langle\langle T_0 \gamma^\mu \partial_\mu \psi \bar{\psi}' \rangle\rangle - \gamma^\mu \partial_\mu \langle\langle T_0 \psi \bar{\psi}' \rangle\rangle = -\delta, \\ \mathcal{O}(\psi', \bar{\psi}\gamma) &:= \langle\langle T_0 \psi' \partial_\mu \bar{\psi} \rangle\rangle \gamma^\mu - \partial_\mu \langle\langle T_0 \psi' \bar{\psi} \rangle\rangle \gamma^\mu = +\delta.\end{aligned}\tag{6.14}$$

Indeed, using (5.2), we obtain

$$\mathcal{O}(\gamma\psi, \bar{\psi}') = -(\not{\partial} + im_\psi) \langle\langle T_0 \psi \bar{\psi}' \rangle\rangle = -i(\not{\partial} + im_\psi) S^F(x - x') = -\delta(x - x'),$$

and the second case follows similarly.

7 Computing the second-order constraints

A priori, in equation (6.3) there may be three kinds of contractions pertinent to our problem of the type (6.7), coming from the *crossing* of the respectively bosonic and fermionic couplings T_1^B and T_1^F with the Q_1^B and Q_1^F vectors. These crossings contain information about the fermionic vertices. Happily, the bosonic interaction set T_1^B and the fermionic Q_1^F -vertex turn out an inert combination, because there are no obstructions involving the form-valued fields w_a .

Our goal in this section is to determine the couplings, as far as possible, from the vanishing of obstructions of type (6.7). Firstly, we seek the \tilde{b}_3 and \tilde{b}_4 coefficients of the Z -boson, which are determined together with the higgs couplings c_0 and c_5 . Secondly, we shall be able to determine the quotient b_1/\tilde{b}_1 , thereby obtaining chirality of the charged boson interactions in the SM; the value of b_1 is trivially determined afterwards. Thirdly, we shall look for the electromagnetic coupling b_5 . At the end, we find the missing terms for the neutral current, and show vanishing of the other higgs couplings.

7.1 Step 1

We start by considering crossings leading to the field content

$$w_Z(x, l) \phi_Z(x, l) \bar{e}(x) e(x).$$

For convenience, we shall omit the factor g^2 in all crossings. One such crossing of the (Q_1^B, T_1^F) -type, from the last term $-\frac{1}{2\cos\Theta} m_Z \partial_\mu \phi_4 \phi_Z w_Z$ in (3.6) with the term $c_0 \phi_4 \bar{e} e$ in (5.5). From the table (6.13), this contributes to the total obstruction the term:

$$-ic_0 \frac{m_Z}{\cos\Theta} w_Z(x, l) \phi_Z(x, l) \bar{e}(x) e(x) \delta(x - x').$$

A factor of 2 comes from appending the identical second contribution in (6.3); we do likewise from now on without further notice.

On the other hand, there is a crossing of type (Q_1^F, T_1^F) , matching $\tilde{b}_3 w_Z \bar{e} \gamma^\mu \gamma^5 e$ in (5.3) and $2im_e \tilde{b}_3 \phi_Z \bar{e} \gamma^5 e$ in (5.5). (For the benefit of the reader, all possible fermionic crossings are

listed in Appendix C.) Here there are two \bar{e} - e contractions of equal value, see the table (C.1), for a total contribution of

$$8im_e\tilde{b}_3^2 w_Z(x,l)\phi_Z(x,l)\bar{e}(x)e(x)\delta(x-x').$$

String independence therefore demands cancellation of the last two expressions; and so

$$c_0 = \frac{8\tilde{b}_3^2 m_e \cos^2 \Theta}{m_W}.$$

Next, we look for crossings with field content $w_Z(x,l)\phi_4(x)\bar{e}(x)\gamma^5 e(x)$. There is a crossing of type (Q_1^B, T_1^F) , of $\frac{1}{2\cos\Theta}m_Z w_Z \phi_4 \partial_\mu \phi_Z$ from (3.6) with $2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e$ from (5.3). Now (6.13) yields

$$-2\tilde{b}_3 \frac{m_e}{m_W} w_Z(x,l)\phi_4(x)\bar{e}(x)\gamma^5 e(x)\delta(x-x').$$

Now there appear to be two relevant (Q_1^F, T_1^F) -type crossings: $\tilde{b}_3 w_Z \bar{e} \gamma^\mu \gamma^5 e$ with $c_0 \phi_4 \bar{e} e$ and $b_3 w_Z \bar{e} \gamma^\mu e$ with $\tilde{c}_0 \phi_4 \bar{e} \gamma^5 e$. The second happens to vanish – see (C.1) again – and the first yields

$$4\tilde{b}_3 c_0 w_Z(x,l)\phi_4(x)\bar{e}(x)\gamma^5 e(x)\delta(x-x').$$

Altogether we arrive at

$$c_0 = \frac{m_e}{2m_W} = \frac{8\tilde{b}_3^2 m_e \cos^2 \Theta}{m_W}, \quad \text{and thus} \quad \tilde{b}_3 = \pm \frac{1}{4\cos\Theta} =: \varepsilon_1 \frac{1}{4\cos\Theta}. \quad (7.1)$$

In much the same way, for the field content $w_Z(x,l)\phi_Z(x,l)\bar{\nu}(x)\nu(x)$, the only crossings are $-\frac{1}{2\cos\Theta}m_Z w_Z \partial_\mu \phi_4 \phi_Z$ with $c_5 \phi_4 \bar{\nu} \nu$ and $\tilde{b}_4 w_Z \bar{\nu} \gamma^\mu \gamma^5 \nu$ with $2im_\nu \tilde{b}_4 \phi_Z \bar{\nu} \gamma^5 \nu$. These cancel provided that

$$c_5 = \frac{8\tilde{b}_4^2 m_\nu \cos^2 \Theta}{m_W}.$$

On the other hand, the field content $w_Z(x,l)\phi_4(x)\bar{\nu}(x)\gamma^5 \nu(x)$ can arise from four crossings: $\frac{1}{2\cos\Theta}m_Z w_Z \phi_4 (\partial_\mu \phi_Z - Z_\mu)$ with both $2im_\nu \tilde{b}_4 \phi_Z \bar{\nu} \gamma^5 \nu$ and $\tilde{b}_4 Z_\mu \bar{\nu} \gamma^\mu \gamma^5 \nu$; and moreover the (Q_1^F, T_1^F) -type ones $\tilde{b}_4 w_Z \bar{\nu} \gamma^\mu \gamma^5 \nu$ with $c_5 \phi_4 \bar{\nu} \nu$, and $b_4 w_Z \bar{\nu} \gamma^\mu \nu$ with $\tilde{c}_5 \phi_4 \bar{\nu} \gamma^5 \nu$. The second and fourth crossings again happen to vanish. Cancellation of the first and third leads to

$$c_5 = \frac{m_\nu}{2m_W} \quad \text{and} \quad \tilde{b}_4 = \pm \frac{1}{4\cos\Theta} =: \varepsilon_2 \frac{1}{4\cos\Theta}. \quad (7.2)$$

Note that the higgs couplings c_0 and c_5 come out respectively proportional to the electron and neutrino masses, with the same proportionality constant – as it should be.⁸

⁸We have left aside the possibility that \tilde{b}_3 , c_0 , \tilde{b}_4 and c_5 all vanish; this will soon be refuted.

7.2 Step 2: the road to chirality

The signs ε_1 and ε_2 turn out to be related. To see that, consider together obstructions of the form $w_-W_{+\kappa}\bar{e}\gamma^\kappa\gamma^5e$ and $w_+W_{-\kappa}\bar{e}\gamma^\kappa\gamma^5e$. They may come from crossings of type (Q_1^F, T_1^F) :

$$\begin{aligned} b_1w_-\bar{e}\gamma^\mu v & \text{ with } \tilde{b}_1W_{+\kappa}\bar{v}\gamma^\kappa\gamma^5e & \text{ and } \tilde{b}_1w_-\bar{e}\gamma^\mu\gamma^5v & \text{ with } b_1W_{+\kappa}\bar{v}\gamma^\kappa e, \\ b_1w_+\bar{v}\gamma^\mu e & \text{ with } \tilde{b}_1W_{-\kappa}\bar{e}\gamma^\kappa\gamma^5v & \text{ and } \tilde{b}_1w_+\bar{v}\gamma^\mu\gamma^5e & \text{ with } b_1W_{-\kappa}\bar{e}\gamma^\kappa v. \end{aligned}$$

Each line gives rise to two identical obstructions, with total value

$$-4b_1\tilde{b}_1(w_-W_{+\kappa} - w_+W_{-\kappa})\bar{e}\gamma^\kappa\gamma^5e\delta(x-x').$$

Such a term also arises from the (Q_1^B, T_1^F) -type crossing of $i\cos\Theta(w_-W_+^\lambda - w_+W_-^\lambda)F_{\mu\lambda}^Z$ in (3.5) with $\tilde{b}_3Z_\kappa\bar{e}\gamma^\kappa\gamma^5e$. As we saw in Section 6, this is a “dangerous” crossing, yielding

$$\begin{aligned} & 2\tilde{b}_3\cos\Theta(w_-W_{+\kappa} - w_+W_{-\kappa})\bar{e}\gamma^\kappa\gamma^5e\delta(x-x') \\ & + 2i\tilde{b}_3\cos\Theta(w_-W_+^\lambda - w_+W_-^\lambda)\bar{e}\gamma^\kappa\gamma^5e\partial^\mu(c_{[\lambda\mu]\kappa}\delta_l(x-x')). \end{aligned}$$

The term $il_\lambda\partial_\kappa\delta_l$ in $\mathcal{O}(F_{\bullet\lambda}^Z, Z'_\kappa)$ does not contribute, since $l_\lambda W_\pm^\lambda = 0$ by transversality (see Section 2).

We obtain in all

$$\begin{aligned} & (2\tilde{b}_3\cos\Theta - 4b_1\tilde{b}_1)(w_-W_{+\kappa} - w_+W_{-\kappa})\bar{e}\gamma^\kappa\gamma^5e\delta(x-x') \\ & - 2i\tilde{b}_3\cos\Theta(w_-W_+^\lambda - w_+W_-^\lambda)\bar{e}\gamma^\kappa\gamma^5e\partial^\mu(c_{[\lambda\mu]\kappa}\delta_l(x-x')). \end{aligned}$$

Here string independence dictates that $c_{[\lambda\mu]\kappa} = 0$. The end result is

$$2b_1\tilde{b}_1 = \tilde{b}_3\cos\Theta. \quad (7.3a)$$

A completely parallel computation, for obstructions of the form $w_\mp W_{\pm\kappa}\bar{v}\gamma^\kappa\gamma^5v$, gives the relation

$$2b_1\tilde{b}_1 = -\tilde{b}_4\cos\Theta. \quad (7.3b)$$

In view of (7.1) and (7.2), this says that $\varepsilon_2 = -\varepsilon_1$.

Now we observe that $w_-\phi_Z\bar{e}v$ is produced either by the term from (3.4) of the form $\frac{i}{2}m_W^2\sec\Theta w_-\partial_\mu\phi_+\phi_Z$, crossed with $i(m_e - m_v)b_1\phi_-\bar{e}v$ from (5.5); or by purely fermionic crossings, between $\tilde{b}_1w_-\bar{e}\gamma^\mu\gamma^5v$ and $2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5e + 2im_v\tilde{b}_4\phi_Z\bar{v}\gamma^5v$. This, together with (7.1) and (7.2), leads to

$$i(m_e - m_v)b_1 = 2\tilde{b}_1(2im_e\tilde{b}_3 + 2im_v\tilde{b}_4)\cos\Theta = i(m_e - m_v)\varepsilon_1\tilde{b}_1.$$

The outcome⁹ is that $\tilde{b}_1 = \varepsilon_1b_1$: **chirality!**

⁹Notice now that were $\tilde{b}_3 = 0$ or $\tilde{b}_4 = 0$, it would follow that $b_1 = \tilde{b}_1 = 0$ too, and the whole term T_1^F would vanish. Thus none of these coefficients are zero, and (7.1) is confirmed, with $c_0 \neq 0$ and $c_5 \neq 0$ as well.

Of course, this procedure does not tell us whether $\varepsilon_1 = +1$ or $\varepsilon_1 = -1$. The second of these appears to be Nature's decision. Equations (7.3) now dictate that $b_1^2 = \tilde{b}_1^2 = 1/8$. This determines b_1 , up to a sign; we choose $b_1 = -1/2\sqrt{2}$.

Observe that the proof of chirality requires the presence of a higgs, at the level of tree graphs. There are several consistency cases for the scalar particle of the Standard Model. But it is hard to think of a simpler one. (We owe this remark to Alejandro Ibarra.)

7.3 Step 3: electric charge

Consider the term $-i \sin \Theta w_- A^\lambda F_{\mu\lambda}^+$ in (3.5), crossed with the term $b_1 W_{-\kappa} \bar{e} \gamma^\kappa v$ in (5.5); and the crossing of $b_1 w_- \bar{e} \gamma^\mu v$ with $b_5 A_\kappa \bar{e} \gamma^\kappa e$. These are the only terms yielding the field content $w_- A_\kappa \bar{e} \gamma^\kappa v$. The total obstruction is

$$(2b_1 b_5 - 2b_1 \sin \Theta) w_- (x, l) A_\kappa (x, l) \bar{e}(x) \gamma^\kappa v(x) \delta(x - x').$$

This vanishes if and only if $b_5 = \sin \Theta$, that is, $g b_5 = g \sin \Theta$: an important tenet of electroweak theory [28].

The case could also have been made from the crossings with field content $w_+ A_\kappa \bar{v} \gamma^\kappa e$, *mutatis mutandis*.

7.4 Step 4: coda

It remains to determine the couplings b_3 and b_4 for the neutral current. For that, we seek first the contributions with content $w_- W_{+\kappa} \bar{e} \gamma^\kappa e$. The crossings are of four classes:

$$\begin{aligned} i \sin \Theta w_- W_+^\lambda F_{\mu\lambda} & \quad \text{with} \quad b_5 A_\kappa \bar{e} \gamma^\kappa e, \\ i \cos \Theta w_- W_+^\lambda F_{\mu\lambda}^Z & \quad \text{with} \quad b_3 Z_\kappa \bar{e} \gamma^\kappa e, \\ b_1 w_- \bar{e} \gamma^\mu v & \quad \text{with} \quad b_1 W_{+\kappa} \bar{v} \gamma^\kappa e, \\ \tilde{b}_1 w_- \bar{e} \gamma^\mu \gamma^5 v & \quad \text{with} \quad \tilde{b}_1 W_{+\kappa} \bar{v} \gamma^\kappa \gamma^5 e. \end{aligned}$$

The cancellation of the total obstruction now entails

$$b_3 \cos \Theta + \sin^2 \Theta = b_1^2 + \tilde{b}_1^2 = \frac{1}{4}, \quad \text{that is,} \quad b_3 = \frac{1}{4 \cos \Theta} - \frac{\sin^2 \Theta}{\cos \Theta}.$$

Similarly, from the crossing of $i \cos \Theta w_- W_+^\lambda F_{\mu\lambda}^Z$ with $b_4 Z_\kappa \bar{v} \gamma^\kappa v$, and the same fermionic terms as before, the contributions with content $w_- W_{+\kappa} \bar{v} \gamma^\kappa v$ cancel only if

$$b_4 \cos \Theta = -b_1^2 - \tilde{b}_1^2 = -\frac{1}{4}, \quad \text{and thus} \quad b_4 = -\frac{1}{4 \cos \Theta}.$$

The expected result of the neutral current containing a right-handed component has been obtained.

Finally, crossing the term $-\frac{1}{2} m_Z \sec \Theta w_Z \phi_Z \partial_\mu \phi_4$ in (3.6) with the terms $\tilde{c}_0 \phi_4 \bar{e} \gamma^5 e$ and $\tilde{c}_5 \phi_4 \bar{v} \gamma^5 v$ of (5.5) gives rise to terms with content $w_Z \phi_Z \bar{e} \gamma^5 e$ and $w_Z \phi_Z \bar{v} \gamma^5 v$, respectively.

The crossings of $b_3 w_Z \bar{e} \gamma^\mu e$ with $2im_e \tilde{b}_3 \phi_Z \bar{e} \gamma^5 e$ and $b_4 w_Z \bar{\nu} \gamma^\mu \nu$ with $2im_\nu \tilde{b}_4 \phi_Z \bar{\nu} \gamma^5 \nu$, respectively, vanish of their own accord: see the table (C.1). Therefore, they cannot cancel the former crossings, and so $\tilde{c}_0 = \tilde{c}_5 = 0$ must hold. That is to say, the couplings of the higgs *are not chiral*.

In conclusion, we exhibit the leptonic coupling (for one family) of the SM, as derived from string-independence. For definiteness, we take $\epsilon_1 = -1$, which is the experimental fact.

$$T_1^F = g \left\{ -\frac{1}{2\sqrt{2}} W_{-\mu} \bar{e} \gamma^\mu (1 - \gamma^5) \nu - \frac{1}{2\sqrt{2}} W_{+\mu} \bar{\nu} \gamma^\mu (1 - \gamma^5) e + \frac{1 - 4\sin^2 \Theta}{4\cos \Theta} Z_\mu \bar{e} \gamma^\mu e \right. \\ - \frac{1}{4\cos \Theta} Z_\mu \bar{e} \gamma^\mu \gamma^5 e - \frac{1}{4\cos \Theta} Z_\mu \bar{\nu} \gamma^\mu (1 - \gamma^5) \nu + \sin \Theta A_\mu \bar{e} \gamma^\mu e \\ + i \frac{m_e - m_\nu}{2\sqrt{2}} (\phi_- \bar{e} \nu - \phi_+ \bar{\nu} e) - i \frac{m_e + m_\nu}{2\sqrt{2}} (\phi_- \bar{e} \gamma^5 \nu + \phi_+ \bar{\nu} \gamma^5 e) \\ \left. - i \frac{m_e}{2\cos \Theta} \phi_Z \bar{e} \gamma^5 e + i \frac{m_\nu}{2\cos \Theta} \phi_Z \bar{\nu} \gamma^5 \nu + \frac{m_e}{2m_W} \phi_4 \bar{e} e + \frac{m_\nu}{2m_W} \phi_4 \bar{\nu} \nu \right\}. \quad (7.4)$$

Amazingly, this differs from what is known from the standard treatment by only by a divergence. Namely, $T_1^F = T_1^{F,p} + (\partial V)$, where $T_1^{F,p}$ is almost pointlike,

$$T_1^{F,p} = g \left\{ -\frac{1}{2\sqrt{2}} W_{-\mu}^p \bar{e} \gamma^\mu (1 - \gamma^5) \nu - \frac{1}{2\sqrt{2}} W_{+\mu}^p \bar{\nu} \gamma^\mu (1 - \gamma^5) e + \frac{1 - 4\sin^2 \Theta}{4\cos \Theta} Z_\mu^p \bar{e} \gamma^\mu e \right. \\ - \frac{1}{4\cos \Theta} Z_\mu^p \bar{e} \gamma^\mu \gamma^5 e - \frac{1}{4\cos \Theta} Z_\mu^p \bar{\nu} \gamma^\mu (1 - \gamma^5) \nu + \sin \Theta A_\mu \bar{e} \gamma^\mu e \\ \left. + \frac{m_e}{2m_W} \phi_4 \bar{e} e + \frac{m_\nu}{2m_W} \phi_4 \bar{\nu} \nu \right\}; \quad (7.5)$$

and the divergence of the expression

$$V^\mu = g \left\{ -\frac{1}{2\sqrt{2}} \phi_- \bar{e} \gamma^\mu (1 - \gamma^5) \nu - \frac{1}{2\sqrt{2}} \phi_+ \bar{\nu} \gamma^\mu (1 - \gamma^5) e + \frac{1 - 4\sin^2 \Theta}{4\cos \Theta} \phi_Z \bar{e} \gamma^\mu e \right. \\ \left. - \frac{1}{4\cos \Theta} \phi_Z \bar{e} \gamma^\mu \gamma^5 e - \frac{1}{4\cos \Theta} \phi_Z \bar{\nu} \gamma^\mu (1 - \gamma^5) \nu \right\}$$

sweeps away the escort fields. We wrote “almost pointlike” because the fields in expression (7.5) are pointlike, except for the photon field A_μ , which remains stringlike. For the good reason that W_\pm and Z can be lodged in a Hilbert space, while A cannot. Incidentally, this causes the interacting electron field to be string-localized, thus making contact with the early literature on stringlike fields [9, 10]. A key observation is that (∂V) is *not renormalizable* by power counting, whereas (∂Q) is.

We rest our case. The only way to disprove it would be to find an inconsistency coming from crossings not discussed so far. To verify that this does not happen is a routine, if utterly tedious, exercise.

A last remark is in order. In the stringlike version of electroweak theory, the eventual need of “renormalizing” the original time-ordered product T_0 , as in (6.10), arises. We only found that the skewsymmetric part of $c_{\lambda\mu\kappa}$ in that formula must vanish. Whether or not the theory requires a time-ordered product different from T_0 remains an open question.

8 Conclusion and outlook

To repeat: interactions of quanta should spring from a simple underlying principle. Gauge field theory has played this unifying role so far. That flows from the embarrassing clash of the positivity axioms of Quantum Mechanics with the convenient description of electromagnetic and other forces in terms of potentials. Not unreasonably, the difficulty was elevated into a principle, and one that put geometry in the saddle. The resulting top-down approach, with the need of “quantizing” the Lagrangian description, has ridden us (without much mercy) for many a year.

It should be recognized, however, that the gauge-plus-BRST-invariance framework is just a very useful theoretical *technology* to grapple with elementary particle physics problems. Other theoretical technologies can, and sometimes are and should be, used to address them. Stringlike field theory is but one of those. Ours is a thoroughly bottom-up approach: nothing but the basic axioms of relativity and quantum mechanics are invoked. With the early dividends that the mentioned clash fades away and unbounded-helicity particles take their due place among quantum fields [11].

There is a hefty price to pay, to be sure, in that the extra variable complicates renormalized perturbation theory and the proof of renormalizability of physical models in general. Notwithstanding, the string-independence principle becomes a powerful guide to interacting models. Internal symmetries are shown as consequences of quantum mechanics in the presence of Lorentz symmetry, and a bottom-up construction of the string-local equivalent for self-interaction Yang–Mills ensues [30].¹⁰

Coming now to SM theory, one realizes that embracing SLF pays other dividends. In consonance with the bottom-up method, only string-independence and the physical spectrum of particles are employed to construct the model; puzzles in the usual procedures, like the shape of the higgs’ potential [29] and the origin of chirality in the vector bosons interactions become explained. That said, the model expounded here is of course anomalous, which manifests itself in T_3 . The cure is the same as in the standard treatments. The computation of the chiral anomaly in our framework will be published elsewhere.

A Integrals of quantum fields along lines

Legitimacy of expressions like (2.2) can be seen by formally applying Cartan’s formula of differential geometry to any Proca field A^P , regarded as a differential 1-form, for any kind of

¹⁰There is nothing really new in this: in the seventies it was generally understood that unitarity and renormalizability requirements impose internal symmetries [37, 38]. For heavy vector boson interactions the Higgs’ mechanism shortcut replaced this wisdom in the textbooks.

string l one has

$$A^P(x) - A^P(x + \lambda l) = - \int_0^\lambda dt [i(l)F(x + tl) + di(l)A^P(x + tl)],$$

so that in the $\lambda \uparrow \infty$ limit we would obtain

$$A^P(x) = A^P(x + \infty l) + A(x, l) - d\phi(x, l),$$

which is (2.5), provided $A^P(x + \infty l)$ vanishes. But is a quantum field vanishingly small at the infinity in any reasonable sense? Yes indeed. Quantum fields are operator-valued *distributions*. As the for latter, in massive models they belong to the convolution algebra \mathcal{D}'_{L^1} of integrable distributions, which vanish at infinity in a weak sense.

B Proof of Eq. (6.6)

We prove here the identities

$$\sum_\phi d_l \frac{\partial T_1}{\partial \phi} \langle\langle T_0 \phi \chi' \rangle\rangle + \frac{\partial T_1}{\partial \phi} \langle\langle T_0(d_l \phi) \chi' \rangle\rangle = [T_0(d_l T_1) \chi']_{\text{tree}}, \quad (\text{B.1})$$

$$\sum_\psi \left(\partial_\mu \frac{\partial Q^\mu}{\partial \psi} \langle\langle T_0 \psi \chi' \rangle\rangle + \frac{\partial Q^\mu}{\partial \psi} \langle\langle T_0(\partial_\mu \psi) \chi' \rangle\rangle \right) = [T_0(\partial_\mu Q^\mu) \chi']_{\text{tree}}. \quad (\text{B.2})$$

Using the identity

$$d_l T_1 = \sum_\phi : \frac{\partial T_1}{\partial \phi} d_l \phi :,$$

the right hand side of Eq. (B.1) is

$$\sum_\phi \left[T_0 : \frac{\partial T_1}{\partial \phi} d_l \phi : \chi' \right]_{\text{tree}} = \sum_\psi \left\{ \sum_\phi : \frac{\partial^2 T_1}{\partial \phi \partial \psi} d_l \phi : \right\} \langle\langle T_0 \psi \chi' \rangle\rangle + \sum_\phi \frac{\partial T_1}{\partial \phi} \langle\langle T_0(d_l \phi) \chi' \rangle\rangle.$$

But the term in braces is just $d_l \frac{\partial T_1}{\partial \psi}$. Hence the right hand side of the above equation coincides with the left hand side of Eq. (B.1).

Similarly, using $\partial_\mu Q^\mu = \sum_\phi : (\partial Q^\mu / \partial \phi) \partial_\mu \phi :$, the right hand side of Eq. (B.2) becomes

$$\sum_\psi \left\{ \sum_\phi : \frac{\partial^2 Q^\mu}{\partial \phi \partial \psi} \partial_\mu \phi : \right\} \langle\langle T_0 \psi \chi' \rangle\rangle + \sum_\phi \frac{\partial Q^\mu}{\partial \phi} \langle\langle T_0(\partial_\mu \phi) \chi' \rangle\rangle,$$

which equals the left hand side of Eq. (B.2).

C Fermionic crossings

The crossings of fermionic type in Section 7 are computed as follows. When crossing $\bar{e}\gamma^\mu v$ with $\bar{v}'\gamma^\kappa\gamma^5 e'$, say, one meets two obstructions of type (6.14): contracting the neutrinos gives a factor $\mathcal{O}(\gamma v, \bar{v}') = -\delta(x-x')$, whereas contraction of the electrons gives $\mathcal{O}(e', \bar{e}\gamma) = +\delta(x-x')$. Thus the overall crossing yields a sum of two terms

$$-\bar{e}(x)\gamma^\kappa\gamma^5 e(x)\delta(x-x') + \bar{v}(x)\gamma^\kappa\gamma^5 v(x)\delta(x-x').$$

On the other hand, crossing of $\bar{e}\gamma^\mu\gamma^5 e$ with $\bar{e}'\gamma^5 e'$, say, involving both $\mathcal{O}(\gamma e, \bar{e}')$ and $\mathcal{O}(e', \bar{e}\gamma)$, gives two equal contributions of $\bar{e}(x)e(x)\delta(x-x')$ to the total obstruction.

There are sixteen kinds of crossings in all, taking account of the order of the contractions, and the presence or absence of γ^κ and/or γ^5 factors. Let f denote a fermion (v or e , as the case may be). When computing the crossings, we label the contracted terms with stars: either $\gamma^\mu f \bar{f}'$ is replaced by $\mathcal{O}(\gamma f, \bar{f}') = -\delta$, or $f' \bar{f}\gamma^\mu$ is replaced by $\mathcal{O}(f', \bar{f}\gamma) = +\delta$. In the table which follows, σ and τ denote uncontracted fermions:

$$\begin{array}{ll}
\bar{\sigma}\gamma^\mu f \bar{f}'\gamma^\kappa\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\tau \cdot \delta, & \bar{f}'\gamma^\mu\tau \bar{\sigma}'\gamma^\kappa f' \rightsquigarrow +\bar{\sigma}\gamma^\kappa\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu\gamma^5 f \bar{f}'\gamma^\kappa\gamma^5\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\tau \cdot \delta, & \bar{f}'\gamma^\mu\gamma^5\tau \bar{\sigma}'\gamma^\kappa\gamma^5 f' \rightsquigarrow +\bar{\sigma}\gamma^\kappa\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu\gamma^5 f \bar{f}'\gamma^\kappa\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\gamma^5\tau \cdot \delta, & \bar{f}'\gamma^\mu\gamma^5\tau \bar{\sigma}'\gamma^\kappa f' \rightsquigarrow +\bar{\sigma}\gamma^\kappa\gamma^5\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu f \bar{f}'\gamma^\kappa\gamma^5\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\gamma^5\tau \cdot \delta, & \bar{f}'\gamma^\mu\tau \bar{\sigma}'\gamma^\kappa\gamma^5 f' \rightsquigarrow +\bar{\sigma}\gamma^\kappa\gamma^5\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu f \bar{f}'\tau' \rightsquigarrow -\bar{\sigma}\tau \cdot \delta, & \bar{f}'\gamma^\mu\tau \bar{\sigma}' f' \rightsquigarrow +\bar{\sigma}\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu\gamma^5 f \bar{f}'\gamma^5\tau' \rightsquigarrow +\bar{\sigma}\tau \cdot \delta, & \bar{f}'\gamma^\mu\gamma^5\tau \bar{\sigma}'\gamma^5 f' \rightsquigarrow +\bar{\sigma}\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu\gamma^5 f \bar{f}'\tau' \rightsquigarrow +\bar{\sigma}\gamma^5\tau \cdot \delta, & \bar{f}'\gamma^\mu\gamma^5\tau \bar{\sigma}' f' \rightsquigarrow +\bar{\sigma}\gamma^5\tau \cdot \delta, \\
\bar{\sigma}\gamma^\mu f \bar{f}'\gamma^5\tau' \rightsquigarrow -\bar{\sigma}\gamma^5\tau \cdot \delta, & \bar{f}'\gamma^\mu\tau \bar{\sigma}'\gamma^5 f' \rightsquigarrow +\bar{\sigma}\gamma^5\tau \cdot \delta.
\end{array} \tag{C.1}$$

D Proof of locality of the stringy fields

We prove here locality in the sense that $A_\mu(x, l)$ and $A_\alpha(x', l')$ commute if the strings $\{x + tl\}$ and $\{x' + tl'\}$ are causally disjoint. We begin with some geometric considerations about wedge regions. These are Poincaré transforms of the wedge

$$W_1 := \{x \in \mathbb{R}^4 : x^1 > |x^0|\}.$$

Associated with W_1 are the one-parameter group $\Lambda_1(\cdot)$ of Lorentz boosts which leave W_1 invariant, and the reflection j_1 across the edge of the wedge. More specifically, $\Lambda_1(t)$ acts as

$$\begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{pmatrix}$$

and j_1 acts as the reflection on the coordinates x^0 and x^1 , leaving the other coordinates unchanged. For a general wedge $W = LW_1 = a + \Lambda W_1$ with $L = (a, \Lambda)$, one defines the corresponding boosts $\Lambda_W(\cdot)$ and reflection j_W by

$$\Lambda_W(t) := L\Lambda_1(t)L^{-1}, \quad j_W := Lj_1L^{-1}.$$

The reflection j_W results from analytic extension of the (entire analytic) matrix-valued function $\Lambda_W(z)$ at $z = i\pi$.

- Lemma 2.** (i) *A string $\{x + tl\}$ is contained in the closure of a wedge $W = a + \Lambda W_1$ if and only if x and l are contained in the closures of W and ΛW_1 respectively.*
- (ii) *Suppose that the strings $\{x + tl\}$ and $\{x' + tl'\}$ are causally disjoint. Then there is a wedge W whose closure contains $\{x + tl\}$ and whose causal complement contains $\{x' + tl'\}$.*

Proof. Item (i) is the same as in Lemma A.1. of [12], whose proof is valid for any direction $l \in \mathbb{R}^4$.

For item (ii), take $W := \frac{1}{2}(x + x') + W_{l,l'}$, where $W_{l,l'} := \{y : (yl) < 0 < (yl')\}$. The causal complement of W is the closure of $\frac{1}{2}(x + x') + W_{l',l}$, see [39].

Using the elementary fact that $\{x + tl\}$ and $\{x' + tl'\}$ are causally disjoint if and only if $(x - x')^2 < 0$ and $((x' - x)l) \geq 0 \geq ((x' - x)l')$, one readily verifies the claim [15]. \square

We now prove locality of the two-point function, recalling first that the on-shell two-point function for not necessarily coinciding directions is given, instead of (2.7), by

$$M_{\mu\nu}^{AA}(p, l, l') = -g_{\mu\nu} + \frac{p_\mu l_\nu}{(pl)} + \frac{p_\nu l'_\mu}{(pl')} - \frac{p_\mu p_\nu (ll')}{(pl)(pl')},$$

see [18]. Given the two strings, let W be a wedge whose closure contains $\{x + tl\}$ and whose causal complement contains $\{x' + tl'\}$ (as in the lemma), and let j_W and $\Lambda_W(t)$ be the reflection and the boosts, respectively, corresponding to W . Denote by g_t the proper non-orthochronous Poincaré transformation $\Lambda_W(-t)j_W$. By translation invariance of the two-point function, we may assume that the edge of W contains the origin. Then x and l are in the closure of W , while x' and l' lie in the causal complement of W . This implies that for t in the strip $\mathbb{R} + i(0, \pi)$ the imaginary parts of $g_t x$, $g_t l$, $g_{-t} x'$ and $g_{-t} l'$ all lie in the closed forward light cone – see, for example, Eq. (A.7) in [12].

Now consider the relation

$$\int d\mu(p) e^{-i(p(x' - g_t x))} M_{\alpha\mu}^{AA}(p, l', g_t l) = \int d\mu(p) e^{-i(p(x - g_{-t} x'))} M_{\alpha\mu}^{AA}(-g_t p, l', g_t l), \quad (\text{D.1})$$

which is verified by applying the transformation $p \mapsto -g_t p$ on the mass shell. (We use $-g_t$ instead of g_t , since the former is an orthochronous Poincaré transformation, while the latter is not orthochronous and maps the positive onto the negative mass shell.) We may write $g_t^{-1} = g_{-t}$, since j_W and $\Lambda_W(t)$ commute. We wish to extend the function $F(t)$ defined

by (D.1) analytically into the strip $\mathbb{R} + i(0, \pi)$. To this end, it is important that the Minkowski products of $g_t x$, $g_t l$, $g_{-t} x'$ and $g_{-t} l'$ with a covector p in the mass shell all have strictly positive imaginary parts due to the remark before Eq. (D.1). This implies firstly that $M_{\alpha\mu}^{AA}(p, l', g_t l)$ extends analytically into the strip, and secondly that $|\exp i(p g_t x)|$ is uniformly bounded by 1 over the strip. Similarly, $M_{\alpha\mu}^{AA}(-g_t p, l', g_t l)$ is analytic and $|\exp i(p g_{-t} x')|$ is bounded by 1. Thus, $F(t)$ has an analytic extension into the strip, and Eq. (D.1) holds, by the Schwarz reflection principle, also at $t = i\pi$. But $g_{\pm i\pi} = 1$, and thus at $t = i\pi$ the left hand side of Eq. (D.1) reduces, up to a factor $(2\pi)^3$, to the vacuum expectation value $\langle\langle A_\alpha(x', l') A_\mu(x, l) \rangle\rangle$. On the right hand side, one verifies that $M_{\alpha\mu}^{AA}(-g_t p, l', g_t l)|_{t=i\pi} = M_{\mu\alpha}^{AA}(p, l, l')$. Thus, at $t = i\pi$ the right hand side of (D.1) reduces, up to a factor $(2\pi)^3$, to $\langle\langle A_\mu(x, l) A_\alpha(x', l') \rangle\rangle$. In short, Eq. (D.1) at $i\pi$ is just the locality of the two-point functions. This implies locality of the fields by a standard argument in the proof of the Jost–Schroer theorem [40].

The analyticity trick employed above has an elegant interpretation in terms of the powerful Bisognano–Wichmann theorem, which has been established to hold in massive theories with localization in spacelike strings [41]. To this end, note that the left and right hand sides of Eq. (D.1) can be interpreted, after multiplication with $(2\pi)^{-3}$ and contracting with $(g_t)^\mu{}_\nu$, as

$$\langle\langle A_\alpha(x', l') U(g_{-t}) A_\nu(x, l)^* \rangle\rangle \quad \text{and} \quad \langle\langle A_\nu(x, l) U(g_t) A_\alpha(x', l')^* \rangle\rangle, \quad (\text{D.2})$$

respectively, where $U(g_t)$ is an antiunitary representer of the non-orthochronous Poincaré transformation g_t . The above-established facts that the left hand side is analytic over the strip and at $t = i\pi$ coincides with $\langle\langle A_\alpha(x', l') A_\nu(x, l) \rangle\rangle$ imply that the vector $A_\nu(x, l)^* \Omega$, where Ω is the vacuum vector, lies in the domain of the unbounded antilinear operator $U(g_{-z})$ for z in the mentioned strip, and $U(g_{-i\pi}) A_\nu(x, l)^* \Omega = A_\nu(x, l) \Omega$. This means just that $U(g_{-i\pi})$ is the Tomita operator for the algebra of fields localized in the wedge W : this is the Bisognano–Wichmann property. Covariance – or, independently, the fact established for the right hand side of (D.1) – implies that $U(g_{i\pi})$ is the Tomita operator for the algebra associated with the causal complement of W , that is, $U(g_{i\pi}) A_\alpha(x', l')^* \Omega = A_\alpha(x', l') \Omega$. The equality in (D.2) is now just a consequence of antiunitarity of $U(g_t)$, invariance of the vacuum vector, and the representation property $U(g_t)^{-1} = U(g_{-t})$, and it implies locality of the two-point function, $\langle\langle A_\alpha(x', l') A_\nu(x, l) \rangle\rangle = \langle\langle A_\nu(x, l) A_\alpha(x', l') \rangle\rangle$.

E A model of leptons

Engineering the GWS model from our formalism is not overly desirable. But we do it here, as promised in the introduction. Let us reconsider the three first lines of expression (7.4). We begin by introducing the notation

$$\Psi_L := \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} := \begin{pmatrix} \frac{1}{2}(1 - \gamma^5) \nu \\ \frac{1}{2}(1 - \gamma^5) e \end{pmatrix}.$$

First,

$$-\frac{1}{\sqrt{2}} W_{-\mu} \bar{e} \gamma^\mu \frac{(1 - \gamma^5)}{2} \nu = -\frac{1}{\sqrt{2}} \bar{\Psi}_L \gamma^\mu \begin{pmatrix} 0 & 0 \\ W_{-\mu} & 0 \end{pmatrix} \Psi_L = -\frac{1}{2} \bar{\Psi}_L \gamma^\mu W_{-\mu} \tau_- \Psi_L;$$

where $\tau_{\pm} = (\tau_1 \pm i\tau_2)/\sqrt{2}$, with τ_i denoting here the Pauli matrices. Similarly,

$$-\frac{1}{\sqrt{2}}W_{+\mu}\bar{\nu}\gamma^{\mu}\frac{(1-\gamma^5)}{2}e = -\frac{1}{2}\bar{\Psi}_L\gamma^{\mu}W_{+\mu}\tau_+\Psi_L.$$

The first two terms in (7.4) are therefore of the form

$$-\frac{1}{2}g\bar{\Psi}_L\gamma^{\mu}(W_{+\mu}\tau_+ + W_{-\mu}\tau_-)\Psi_L = -\frac{1}{2}g\bar{\Psi}_L\gamma^{\mu}(W_{1\mu}\tau_1 + W_{2\mu}\tau_2)\Psi_L. \quad (\text{E.1})$$

Knowing, as we know, that the interaction is governed by a $U(2)$ symmetry, it is tempting to regard ν and e as isospin components, respectively valued $+\frac{1}{2}$ and $-\frac{1}{2}$. The “right-handed leptons” $e_R := \frac{1}{2}(1+\gamma^5)e$ and $\nu_R := \frac{1}{2}(1+\gamma^5)\nu$ are isospin singlets.

Denote by Q the electric charge, so that $Q(e) = -1$ and $Q(\nu) = 0$, and isospin by I_3 . Observe that, putting $\Psi = \Psi_L + \Psi_R$, the next four terms of (7.4) are rendered into:

$$-g\sin\Theta\bar{\Psi}\gamma^{\mu}(A_{\mu} - Z_{\mu}\tan\Theta)Q\Psi - \frac{g}{\cos\Theta}\bar{\Psi}_L\gamma^{\mu}Z_{\mu}I_3\Psi_L. \quad (\text{E.2})$$

In order to translate this into the received framework, with its “covariant gauge transformation” technology, we now introduce the unobservable fields

$$\begin{aligned} W_{3\mu} &:= \cos\Theta Z_{\mu} + \sin\Theta A_{\mu} \\ B_{\mu} &:= -\sin\Theta Z_{\mu} + \cos\Theta A_{\mu} \end{aligned} \quad \text{with inversion} \quad \begin{aligned} A_{\mu} &= \cos\Theta B_{\mu} + \sin\Theta W_{3\mu} \\ Z_{\mu} &= -\sin\Theta B_{\mu} + \cos\Theta W_{3\mu}. \end{aligned}$$

Then, with $g_B := g\tan\Theta$, we can rewrite (E.2) as

$$-g_B\bar{\Psi}\gamma^{\mu}B_{\mu}Q\Psi + g_B\bar{\Psi}_L\gamma^{\mu}B_{\mu}I_3\Psi_L - \frac{1}{2}g\bar{\Psi}_L\gamma^{\mu}W_{3\mu}\tau_3\Psi_L. \quad (\text{E.3})$$

One can now bring in the convention

$$Y = 2(Q - I_3), \quad \text{that is: } Y(e_L) = Y(\nu_L) = -1; Y(e_R) = -2, Y(\nu_R) = 0.$$

Then the first two summands in (E.3) are rewritten as $-\frac{1}{2}g_B\bar{\Psi}\gamma^{\mu}B_{\mu}Y\Psi$; while the last one together with the right hand side of (E.1) yields $-\frac{1}{2}g\bar{\Psi}_L(\gamma^{\mu}\mathbb{W}_{\mu} \cdot \tau)\Psi_L$.

In fine, we have manufactured the interaction parts of the GWS Lagrangian.

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